

Double-lepton polarization asymmetries in $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay

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Abstract. Double-lepton polarization asymmetries in $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay are calculated using a general, model independent form of the effective Hamiltonian. The sensitivities of these asymmetries to the new Wilson coefficients are studied in detail. Furthermore, the correlations between averaged double-lepton polarization asymmetry and branching ratio are analyzed. It is shown that there exist certain regions of the new Wilson coefficients where new physics can be established by measuring the double-lepton polarization asymmetries only.

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1 Introduction

Rare B -decays induced by the flavor-changing neutral current (FCNC) $b \rightarrow s(d)\ell^+\ell^-$ have received a lot of theoretical interest [1]. These transitions provide an important consistency check of the standard model (SM) at loop level, since FCNC transitions are forbidden in the SM at tree level. These decays induced by the FCNC are very sensitive to the new physics beyond the SM. New physics appears in rare decays through the Wilson coefficients which can take values different from their SM counterpart or through the new operator structures in an effective Hamiltonian.

Among the hadronic, leptonic and semileptonic decays, the last decay channels are very significant, since they are theoretically more or less clean, and they have a relatively larger branching ratio. The semileptonic decay channels are described by the $b \rightarrow s(d)\ell^+\ell^-$ transition and they contain many observables like forward–backward asymmetry \mathcal{A}_{FB} , lepton polarization asymmetries, etc. The existence of these observables is very useful and can serve as a testing ground for the standard model (SM) and for looking for new physics beyond the SM. For this reason, many processes, like $B \rightarrow \pi(\rho)\ell^+\ell^-$ [2], $B \rightarrow \ell^+\ell^-\gamma$ [3], $B \rightarrow K\ell^+\ell^-$ [4] and $B \rightarrow K^*\ell^+\ell^-$ [5–12] have been studied comprehensively.

Recently, BELLE and BaBar Collaborations announced the following results for the branching ratios of the $B \rightarrow K^*\ell^+\ell^-$ and $B \rightarrow K\ell^+\ell^-$ decays:

$$\begin{aligned} \mathcal{B}(B \rightarrow K^*\ell^+\ell^-) &= \begin{cases} (11.5_{-2.4}^{+2.6} \pm 0.8 \pm 0.2) \times 10^{-7} & [13], \\ (0.88_{-0.29}^{+0.33}) \times 10^{-6} & [14], \end{cases} \end{aligned}$$

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$$\begin{aligned} \mathcal{B}(B \rightarrow K\ell^+\ell^-) &= \begin{cases} (4.8_{-0.9}^{+1.0} \pm 0.3 \pm 0.1) \times 10^{-7} & [13], \\ (0.65_{-0.13}^{+0.14} \pm 0.04) \times 10^{-6} & [14]. \end{cases} \end{aligned}$$

Another exclusive decay which is described at the inclusive level by the $b \rightarrow s\ell^+\ell^-$ transition is the baryonic $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$ decay. Unlike mesonic decays, the baryonic decays could maintain the helicity structure of the effective Hamiltonian for the $b \rightarrow s$ transition [15].

Many experimentally measurable quantities such as the branching ratio [16], Λ polarization and single-lepton polarization have already been studied in [17,18], respectively. The analysis of such quantities can be useful for a more precise determination of the SM parameters and looking for new physics beyond the SM. It has been pointed out in [19] that some of the single-lepton polarization asymmetries may be too small to be observed and hence may not provide a sufficient number of observables in checking the structure of the effective Hamiltonian. In need of more observables, London et al. [19] take into account the polarizations of both leptons, which are simultaneously measured, and construct a maximum number of independent observables. It should be noted that the forward–backward asymmetries due to the double-lepton polarizations in the $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$ decay are investigated in [20].

In the present work we analyze the possibility of searching for new physics in the baryonic $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$ decay by studying the double-lepton polarization asymmetries, using a general form of the effective Hamiltonian, including all possible forms of interactions. Note that the dependence of the polarized forward–backward asymmetry on the new Wilson coefficients for the meson \rightarrow meson transition has been investigated in [11,20]. In the same manner, an analogous analysis can be carried out to investigate the

sensitivity of the double-lepton polarization asymmetries to the new Wilson coefficients, in the case of the baryon \rightarrow baryon transition.

This paper is organized as follows. In Sect. 2, using the general, model independent form of the effective Hamiltonian, the matrix element for the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ is obtained. In Sect. 3 we calculate the double-lepton polarization asymmetries. Section 4 is devoted to the numerical analysis, discussions and conclusions.

2 Matrix element for the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay

In this section we derive the matrix element for the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay using the general, model independent form of the effective Hamiltonian. At the quark level, the matrix element of the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is described by the $b \rightarrow s \ell^+ \ell^-$ transition. The effective Hamiltonian for the $b \rightarrow s \ell^+ \ell^-$ transition can be written in terms of twelve model independent four-Fermi interactions as [6]

$$\begin{aligned} \mathcal{M} = & \frac{G\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \quad (1) \\ & \times \left\{ C_{SL} \bar{s}_R i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_L \bar{\ell} \gamma^\mu \ell + C_{BR} \bar{s}_L i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R \bar{\ell} \gamma^\mu \ell \right. \\ & + C_{LL}^{\text{tot}} \bar{s}_L \gamma_\mu b_L \bar{\ell} \gamma^\mu \ell_L + C_{LR}^{\text{tot}} \bar{s}_L \gamma_\mu b_L \bar{\ell} \gamma^\mu \ell_R \\ & + C_{RL} \bar{s}_R \gamma_\mu b_R \bar{\ell} \gamma^\mu \ell_L + C_{RR} \bar{s}_R \gamma_\mu b_R \bar{\ell} \gamma^\mu \ell_R \\ & + C_{LRLR} \bar{s}_L b_R \bar{\ell} \gamma^\mu \ell_R + C_{RLLR} \bar{s}_R b_L \bar{\ell} \gamma^\mu \ell_L \\ & + C_{LRRL} \bar{s}_L b_R \bar{\ell} \gamma^\mu \ell_L + C_{RLRL} \bar{s}_R b_L \bar{\ell} \gamma^\mu \ell_R \\ & \left. + C_T \bar{s} \sigma_{\mu\nu} b \bar{\ell} \sigma^{\mu\nu} \ell + i C_{TE} \epsilon_{\mu\nu\alpha\beta} \bar{s} \sigma^{\mu\nu} b \bar{\ell} \sigma^{\alpha\beta} \ell \right\}, \end{aligned}$$

where $q = P_{\Lambda_b} - P_\Lambda = p_1 + p_2$ is the momentum transfer and C_X are the coefficients of the four-Fermi interactions, $L = (1 - \gamma_5)/2$ and $R = (1 + \gamma_5)/2$. The terms with coefficients C_{SL} and C_{BR} describe the penguin contributions, which correspond to $-2m_s C_7^{\text{eff}}$ and $-2m_b C_7^{\text{eff}}$ in the SM, respectively. The next four terms in (1) with coefficients C_{LL}^{tot} , C_{LR}^{tot} , C_{RL} and C_{RR} describe vector type interactions, two (C_{LL}^{tot} and C_{LR}^{tot}) of which contain SM contributions in the form $C_9^{\text{eff}} - C_{10}$ and $C_9^{\text{eff}} - C_{10}$, respectively. Thus, C_{LL}^{tot} and C_{LR}^{tot} can be written as

$$\begin{aligned} C_{LL}^{\text{tot}} &= C_9^{\text{eff}} - C_{10} + C_{LL}, \\ C_{LR}^{\text{tot}} &= C_9^{\text{eff}} + C_{10} + C_{LR}, \end{aligned} \quad (2)$$

where C_{LL} and C_{LR} describe the contributions of new physics. Additionally, (1) contains four scalar type interactions (C_{LRLR} , C_{RLLR} , C_{LRRL} and C_{RLRL}), and two tensor type interactions (C_T and C_{TE}).

The amplitude of the exclusive $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is obtained by calculating the matrix element of \mathcal{H}_{eff} for the $b \rightarrow s \ell^+ \ell^-$ transition between initial and final baryon states $\langle \Lambda | \mathcal{H}_{\text{eff}} | \Lambda_b \rangle$. It follows from (1) that the matrix elements

$$\langle \Lambda | \bar{s} \gamma_\mu (1 \mp \gamma_5) b | \Lambda_b \rangle,$$

$$\langle \Lambda | \bar{s} \sigma_{\mu\nu} (1 \mp \gamma_5) b | \Lambda_b \rangle,$$

$$\langle \Lambda | \bar{s} (1 \mp \gamma_5) b | \Lambda_b \rangle$$

are needed in order to calculate the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay amplitude.

These matrix elements parametrized in terms of the form factors are as follows [17, 21]:

$$\langle \Lambda | \bar{s} \gamma_\mu b | \Lambda_b \rangle = \bar{u}_\Lambda [f_1 \gamma_\mu + i f_2 \sigma_{\mu\nu} q^\nu + f_3 q_\mu] u_{\Lambda_b}, \quad (3)$$

$$\langle \Lambda | \bar{s} \gamma_\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda [g_1 \gamma_\mu \gamma_5 + i g_2 \sigma_{\mu\nu} \gamma_5 q^\nu + g_3 q_\mu \gamma_5] u_{\Lambda_b}, \quad (4)$$

$$\begin{aligned} \langle \Lambda | \bar{s} \sigma_{\mu\nu} b | \Lambda_b \rangle &= \bar{u}_\Lambda [f_T \sigma_{\mu\nu} - i f_T^V (\gamma_\mu q^\nu - \gamma_\nu q^\mu) \\ &\quad - i f_T^S (P_\mu q^\nu - P_\nu q^\mu)] u_{\Lambda_b}, \end{aligned} \quad (5)$$

$$\begin{aligned} \langle \Lambda | \bar{s} \sigma_{\mu\nu} \gamma_5 b | \Lambda_b \rangle &= \bar{u}_\Lambda [g_T \sigma_{\mu\nu} - i g_T^V (\gamma_\mu q^\nu - \gamma_\nu q^\mu) \\ &\quad - i g_T^S (P_\mu q^\nu - P_\nu q^\mu)] \gamma_5 u_{\Lambda_b}, \end{aligned} \quad (6)$$

where $P = p_{\Lambda_b} + p_\Lambda$ and $q = p_{\Lambda_b} - p_\Lambda$.

The form factors of the magnetic dipole operators are defined as

$$\begin{aligned} \langle \Lambda | \bar{s} i \sigma_{\mu\nu} q^\nu b | \Lambda_b \rangle &= \bar{u}_\Lambda [f_1^T \gamma_\mu + i f_2^T \sigma_{\mu\nu} q^\nu + f_3^T q_\mu] u_{\Lambda_b}, \\ \langle \Lambda | \bar{s} i \sigma_{\mu\nu} \gamma_5 q^\nu b | \Lambda_b \rangle &= \bar{u}_\Lambda [g_1^T \gamma_\mu \gamma_5 + i g_2^T \sigma_{\mu\nu} \gamma_5 q^\nu + g_3^T q_\mu \gamma_5] u_{\Lambda_b}. \end{aligned} \quad (7)$$

Using the identity

$$\sigma_{\mu\nu} \gamma_5 = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta},$$

and (5), the last expression in (7) can be written as

$$\langle \Lambda | \bar{s} i \sigma_{\mu\nu} \gamma_5 q^\nu b | \Lambda_b \rangle = \bar{u}_\Lambda [f_T i \sigma_{\mu\nu} \gamma_5 q^\nu] u_{\Lambda_b}.$$

Multiplying (5) and (6) by $i q^\nu$ and comparing with (7), one can easily obtain the following relations between the form factors:

$$\begin{aligned} f_2^T &= f_T + f_T^S q^2, \\ f_1^T &= [f_T^V + f_T^S (m_{\Lambda_b} + m_\Lambda)] q^2 = -\frac{q^2}{m_{\Lambda_b} - m_\Lambda} f_3^T, \\ g_2^T &= g_T + g_T^S q^2, \\ g_1^T &= [g_T^V - g_T^S (m_{\Lambda_b} - m_\Lambda)] q^2 = \frac{q^2}{m_{\Lambda_b} + m_\Lambda} g_3^T. \end{aligned} \quad (8)$$

The matrix element of the scalar $\bar{s} b$ and pseudoscalar $\bar{s} \gamma_5 b$ operators can be obtained from (3) and (4) by multiplying both sides to q^μ and using the equation of motion. Neglecting the mass of the strange quark, we get

$$\langle \Lambda | \bar{s} b | \Lambda_b \rangle = \frac{1}{m_b} \bar{u}_\Lambda [f_1 (m_{\Lambda_b} - m_\Lambda) + f_3 q^2] u_{\Lambda_b}, \quad (9)$$

$$\langle \Lambda | \bar{s} \gamma_5 b | \Lambda_b \rangle = \frac{1}{m_b} \bar{u}_\Lambda [g_1 (m_{\Lambda_b} + m_\Lambda) \gamma_5 - g_3 q^2 \gamma_5] u_{\Lambda_b}. \quad (10)$$

Using these definitions of the form factors, for the matrix element of the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ we get [17, 18]

$$\begin{aligned} \mathcal{M} = & \frac{G\alpha}{4\sqrt{2}\pi} \\ & \times V_{tb}V_{ts}^* \{ \bar{\ell} \gamma^\mu \ell \bar{u}_\Lambda [A_1 \gamma_\mu (1 + \gamma_5) + B_1 \gamma_\mu (1 - \gamma_5)] \\ & + i\sigma_{\mu\nu} q^\nu [A_2 (1 + \gamma_5) + B_2 (1 - \gamma_5)] \\ & + q_\mu [A_3 (1 + \gamma_5) + B_3 (1 - \gamma_5)] u_{\Lambda_b} \\ & + \bar{\ell} \gamma^\mu \gamma_5 \ell \bar{u}_\Lambda [D_1 \gamma_\mu (1 + \gamma_5) + E_1 \gamma_\mu (1 - \gamma_5)] \\ & + i\sigma_{\mu\nu} q^\nu [D_2 (1 + \gamma_5) + E_2 (1 - \gamma_5)] \\ & + q_\mu [D_3 (1 + \gamma_5) + E_3 (1 - \gamma_5)] u_{\Lambda_b} \\ & + \bar{\ell} \ell \bar{u}_\Lambda (N_1 + H_1 \gamma_5) u_{\Lambda_b} + \bar{\ell} \gamma_5 \ell \bar{u}_\Lambda (N_2 + H_2 \gamma_5) u_{\Lambda_b} \\ & + 4C_T \bar{\ell} \sigma^{\mu\nu} \ell \bar{u}_\Lambda [f_T \sigma_{\mu\nu} - i f_T^V (q_\nu \gamma_\mu - q_\mu \gamma_\nu) \\ & - i f_T^S (P_\mu q_\nu - P_\nu q_\mu)] u_{\Lambda_b} \\ & + 4C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{\ell} \sigma_{\alpha\beta} \ell i \bar{u}_\Lambda [f_T \sigma_{\mu\nu} - i f_T^V (q_\nu \gamma_\mu - q_\mu \gamma_\nu) \\ & - i f_T^S (P_\mu q_\nu - P_\nu q_\mu)] u_{\Lambda_b} \}, \end{aligned} \quad (11)$$

where the explicit forms of the functions A_i , B_i , D_i , E_i , H_j and N_j ($i = 1, 2, 3$ and $j = 1, 2$) can be written as [17]

$$\begin{aligned} A_1 = & \frac{1}{q^2} (f_1^T - g_1^T) C_{SL} + \frac{1}{q^2} (f_1^T + g_1^T) C_{BR} \\ & + \frac{1}{2} (f_1 - g_1) (C_{LL}^{\text{tot}} + C_{LR}^{\text{tot}}) \\ & + \frac{1}{2} (f_1 + g_1) (C_{RL} + C_{RR}), \\ A_2 = & A_1 (1 \rightarrow 2), \\ A_3 = & A_1 (1 \rightarrow 3), \\ B_1 = & A_1 (g_1 \rightarrow -g_1; g_1^T \rightarrow -g_1^T), \\ B_2 = & B_1 (1 \rightarrow 2), \\ B_3 = & B_1 (1 \rightarrow 3), \\ D_1 = & \frac{1}{2} (C_{RR} - C_{RL}) (f_1 + g_1) \\ & + \frac{1}{2} (C_{LR}^{\text{tot}} - C_{LL}^{\text{tot}}) (f_1 - g_1), \\ D_2 = & D_1 (1 \rightarrow 2), \\ D_3 = & D_1 (1 \rightarrow 3), \\ E_1 = & D_1 (g_1 \rightarrow -g_1), \\ E_2 = & E_1 (1 \rightarrow 2), \\ E_3 = & E_1 (1 \rightarrow 3), \\ N_1 = & \frac{1}{m_b} (f_1 (m_{\Lambda_b} - m_\Lambda) + f_3 q^2) \\ & \times (C_{LRLR} + C_{RLLR} + C_{LRRL} + C_{RLRL}), \\ N_2 = & N_1 (C_{LRRL} \rightarrow -C_{LRRL}; C_{RLRL} \rightarrow -C_{RLRL}), \\ H_1 = & \frac{1}{m_b} (g_1 (m_{\Lambda_b} + m_\Lambda) - g_3 q^2) \end{aligned} \quad (12)$$

$$\begin{aligned} & \times (C_{LRLR} - C_{RLLR} + C_{LRRL} - C_{RLRL}), \\ H_2 = & H_1 (C_{LRRL} \rightarrow -C_{LRRL}; C_{RLRL} \rightarrow -C_{RLRL}). \end{aligned}$$

From these expressions it follows that $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is described in terms of many form factors. It is shown in [22] that HQET reduces the number of independent form factors to two (F_1 and F_2) irrelevant of the Dirac structure of the corresponding operators, i.e.,

$$\langle \Lambda(p_\Lambda) | \bar{s} \Gamma b | \Lambda(p_{\Lambda_b}) \rangle = \bar{u}_\Lambda [F_1(q^2) + \not{v} F_2(q^2)] \Gamma u_{\Lambda_b}, \quad (13)$$

where Γ is an arbitrary Dirac structure and $v^\mu = p_{\Lambda_b}^\mu / m_{\Lambda_b}$ is the four-velocity of Λ_b . Comparing the general form of the form factors given in (3)–(10) with (13), one can easily obtain the following relations among them (see also [17, 18, 21]):

$$\begin{aligned} g_1 = f_1 = f_2^T = g_2^T = & F_1 + \sqrt{\hat{r}_\Lambda} F_2, \\ g_2 = f_2 = g_3 = f_3 = g_T^V = f_T^V = & \frac{F_2}{m_{\Lambda_b}}, \\ g_T^S = f_T^S = & 0, \\ g_1^T = f_1^T = & \frac{F_2}{m_{\Lambda_b}} q^2, \\ g_3^T = \frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} + m_\Lambda), \\ f_3^T = -\frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} - m_\Lambda), \end{aligned} \quad (14)$$

where $\hat{r}_\Lambda = m_\Lambda^2 / m_{\Lambda_b}^2$.

From (11), we get for the unpolarized decay width

$$\begin{aligned} \left(\frac{d\Gamma}{d\hat{s}} \right)_0 = & \frac{G^2 \alpha^2}{8192 \pi^5} |V_{tb} V_{ts}^*|^2 \lambda^{1/2}(1, \hat{r}_\Lambda, \hat{s}) \\ & \times v \left[\mathcal{T}_0(\hat{s}) + \frac{1}{3} \mathcal{T}_2(\hat{s}) \right], \end{aligned} \quad (15)$$

where $\lambda(1, \hat{r}_\Lambda, \hat{s}) = 1 + \hat{r}_\Lambda^2 + \hat{s}^2 - 2\hat{r}_\Lambda - 2\hat{s} - 2\hat{r}_\Lambda \hat{s}$ is the triangle function, $\hat{s} = q^2 / m_{\Lambda_b}^2$ and $v = \sqrt{1 - 4\hat{m}_\ell^2 / \hat{s}}$ is the lepton velocity, with $\hat{m}_\ell = m_\ell / m_{\Lambda_b}$. The explicit expressions for \mathcal{T}_0 and \mathcal{T}_2 can be found in [18]. In the next section, we present the expressions for the double-lepton polarization asymmetries.

3 Double-lepton polarization asymmetries in the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay

In the present section we calculate the double-lepton polarization asymmetries, i.e., when polarizations of both leptons are considered. In order to calculate the double-lepton polarization asymmetries, we define the following orthogonal unit vectors $s_i^{\pm\mu}$ in the rest frame of ℓ^\pm ($i = L, T$ or N , stand for longitudinal, transversal or normal polarizations, respectively):

$$s_L^{-\mu} = (0, \mathbf{e}_L^-) = \left(0, \frac{\mathbf{p}_-}{|\mathbf{p}_-|} \right),$$

$$\begin{aligned}
s_N^{-\mu} &= (0, \mathbf{e}_N^-) = \left(0, \frac{\mathbf{p}_A \times \mathbf{p}_-}{|\mathbf{p}_A \times \mathbf{p}_-|}\right), \\
s_T^{-\mu} &= (0, \mathbf{e}_T^-) = (0, \mathbf{e}_N^- \times \mathbf{e}_L^-), \\
s_L^{+\mu} &= (0, \mathbf{e}_L^+) = \left(0, \frac{\mathbf{p}_+}{|\mathbf{p}_+|}\right), \\
s_N^{+\mu} &= (0, \mathbf{e}_N^+) = \left(0, \frac{\mathbf{p}_A \times \mathbf{p}_+}{|\mathbf{p}_A \times \mathbf{p}_+|}\right), \\
s_T^{+\mu} &= (0, \mathbf{e}_T^+) = (0, \mathbf{e}_N^+ \times \mathbf{e}_L^+), \quad (16)
\end{aligned}$$

where \mathbf{p}_\mp and \mathbf{p}_A are the three-momenta of the leptons ℓ^\mp and Λ baryon in the center of mass frame (CM) of $\ell^- \ell^+$ system, respectively. Transformation of unit vectors from the rest frame of the leptons to CM frame of leptons can be accomplished by the Lorentz boost. Boosting of the longitudinal unit vectors $s_L^{\pm\mu}$ yields

$$(s_L^{\mp\mu})_{\text{CM}} = \left(\frac{|\mathbf{p}_\mp|}{m_\ell}, \frac{E_\ell \mathbf{p}_\mp}{m_\ell |\mathbf{p}_\mp|}\right), \quad (17)$$

where $\mathbf{p}_+ = -\mathbf{p}_-$, E_ℓ and m_ℓ are the energy and mass of leptons in the CM frame, respectively. The remaining two unit vectors $s_N^{\pm\mu}$, $s_T^{\pm\mu}$ are unchanged under a Lorentz boost.

Having obtained the above expressions, we now define the double-polarization asymmetries as follows [19]:

$$\begin{aligned}
&P_{ij}(q^2) \\
&= \left\{ \left(\frac{d\Gamma(\mathbf{s}_i^-, \mathbf{s}_j^+)}{dq^2} - \frac{d\Gamma(-\mathbf{s}_i^-, \mathbf{s}_j^+)}{dq^2} \right) \right. \\
&\quad \left. - \left(\frac{d\Gamma(\mathbf{s}_i^-, -\mathbf{s}_j^+)}{dq^2} - \frac{d\Gamma(-\mathbf{s}_i^-, -\mathbf{s}_j^+)}{dq^2} \right) \right\} \\
&/ \left\{ \left(\frac{d\Gamma(\mathbf{s}_i^-, \mathbf{s}_j^+)}{dq^2} + \frac{d\Gamma(-\mathbf{s}_i^-, \mathbf{s}_j^+)}{dq^2} \right) \right. \\
&\quad \left. + \left(\frac{d\Gamma(\mathbf{s}_i^-, -\mathbf{s}_j^+)}{dq^2} + \frac{d\Gamma(-\mathbf{s}_i^-, -\mathbf{s}_j^+)}{dq^2} \right) \right\}, \quad (18)
\end{aligned}$$

where the first subindex i represents a lepton and the second one an antilepton. Using this definition of P_{ij} , nine double-lepton polarization asymmetries are calculated. Their expressions are

$$\begin{aligned}
P_{LL} &= \frac{8m_{A_b}^4}{3\Delta} \text{Re} \left\{ 32m_{A_b}^2 \hat{m}_\ell \lambda \left(1 + \sqrt{\hat{r}_\Lambda}\right) (A_1 + B_1) C_T^* f_T^{S*} \right. \\
&\quad - 12 \left(1 + \sqrt{\hat{r}_\Lambda}\right) \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s}\right) \\
&\quad \times \{\hat{m}_\ell (D_1 - E_1) H_2^* \\
&\quad + 8m_{A_b}^2 \hat{m}_\ell \hat{s} (A_2 + B_2) C_T f_T^{V*} + 32m_{A_b} \hat{s} \\
&\quad \times [|C_T|^2 (2v^2 - 1) + 4v^2 |C_{TE}|^2] f_T f_T^{V*} \} \\
&\quad + 12\hat{m}_\ell \left(1 - \sqrt{\hat{r}_\Lambda}\right) \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s}\right) \\
&\quad \times (D_1 + E_1) F_2^*
\end{aligned}$$

$$\begin{aligned}
&- 32m_{A_b}^3 \hat{m}_\ell \lambda \hat{s} (A_2 + B_2) C_T^* f_T^{S*} \\
&\quad + 3\hat{s} \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s}\right) \\
&\quad \times \left[v^2 |F_1|^2 + |F_2|^2 + 4m_{A_b} \hat{m}_\ell (D_3 + E_3) F_2^* \right] \\
&\quad - 12m_{A_b} \sqrt{\hat{r}_\Lambda} (1 - \hat{r}_\Lambda + \hat{s}) \\
&\quad \times [\hat{s} (1 + v^2) (A_1 A_2^* + B_1 B_2^*) - 4\hat{m}_\ell^2 (D_1 D_3^* + E_1 E_3^*)] \\
&\quad + 3\hat{s} \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s}\right) \\
&\quad \times \left[4m_{A_b} \hat{m}_\ell (D_3 - E_3) H_2^* + v^2 |H_1|^2 + |H_2|^2 \right] \\
&\quad + 12m_{A_b} (1 - \hat{r}_\Lambda - \hat{s}) \\
&\quad \times [\hat{s} (1 + v^2) (A_1 B_2^* + A_2 B_1^*) + 4\hat{m}_\ell^2 (D_1 E_3^* + D_3 E_1^*)] \\
&\quad + 24\sqrt{\hat{r}_\Lambda} \hat{s} (1 + v^2) (A_1 B_1^* + D_1 E_1^* + m_{A_b}^2 \hat{s} A_2 B_2^*) \\
&\quad + 24m_{A_b}^2 \hat{m}_\ell^2 \hat{s} (1 + \hat{r}_\Lambda - \hat{s}) (|D_3|^2 + |E_3^*|^2) \\
&\quad + 16m_{A_b} \left[(1 - \hat{r}_\Lambda)^2 + \hat{s} (1 - 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda) - 2\hat{s}^2 \right] \\
&\quad \times [2\hat{m}_\ell (A_1 + B_1) C_T^* f_T^{V*} \\
&\quad - 32m_{A_b} \hat{m}_\ell^2 (|C_T|^2 + 2|C_{TE}|^2) |f_T^{V*}|^2 \\
&\quad + 4m_{A_b} \hat{s} (|C_T|^2 + 4|C_{TE}|^2) |f_T^{V*}|^2] \\
&\quad - 2(1 + v^2) [1 + \hat{r}_\Lambda^2 - \hat{r}_\Lambda (2 - \hat{s}) + \hat{s} (1 - 2\hat{s})] \\
&\quad \times (|A_1|^2 + |B_1|^2) \\
&\quad - 2 [(5v^2 - 3)(1 - \hat{r}_\Lambda)^2 + 4\hat{m}_\ell^2 (1 + \hat{r}_\Lambda) \\
&\quad + 2\hat{s} (1 + 8\hat{m}_\ell^2 + \hat{r}_\Lambda) - 4\hat{s}^2] (|D_1|^2 + |E_1|^2) \\
&\quad - 2m_{A_b}^2 (1 + v^2) \hat{s} [2 + 2\hat{r}_\Lambda^2 - \hat{s} (1 + \hat{s}) - \hat{r}_\Lambda (4 + \hat{s})] \\
&\quad \times (|A_2|^2 + |B_2|^2) \\
&\quad + 64m_{A_b}^3 \lambda \hat{s} \\
&\quad \times [2 (1 + \sqrt{\hat{r}_\Lambda}) f_T^S f_T^{V*} \\
&\quad + m_{A_b} (1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s}) |f_T^S|^2] \\
&\quad \times [(2v^2 - 1) |C_T|^2 + 4v^2 |C_{TE}|^2] \\
&\quad - 4m_{A_b}^2 \hat{s} v^2 [2(1 + \hat{r}_\Lambda^2) - \hat{s} (1 + \hat{s}) - \hat{r}_\Lambda (4 + \hat{s})] \\
&\quad \times (|D_2|^2 + |E_2|^2) \\
&\quad + 24m_{A_b} \hat{s} (1 - \hat{r}_\Lambda - \hat{s}) v^2 (D_1 E_2^* + D_2 E_1^*) \\
&\quad - 24m_{A_b} \sqrt{\hat{r}_\Lambda} \hat{s} (1 - \hat{r}_\Lambda + \hat{s}) v^2 (D_1 D_2^* + E_1 E_2^*) \\
&\quad + 48m_{A_b}^2 \sqrt{\hat{r}_\Lambda} \hat{s} (\hat{s} v^2 D_2 E_2^* + 2\hat{m}_\ell^2 D_3 E_3^*) \\
&\quad - 128m_{A_b}^2 \lambda \hat{s} [(2v^2 - 1) |C_T|^2 + 4v^2 |C_{TE}|^2] f_T f_T^{S*} \\
&\quad + 32m_{A_b} \hat{m}_\ell \{ [2(1 - \hat{r}_\Lambda)^2 - \hat{s} (1 + \hat{r}_\Lambda) - \hat{s}^2]
\end{aligned}$$

$$\begin{aligned}
& \times (C_T - 2C_{TE}) - 6\sqrt{\hat{r}_A}\hat{s} \\
& \times (C_T + 2C_{TE})\} A_2^* f_T \\
& + 32m_{A_b}\hat{m}_\ell \{ [2(1 - \hat{r}_A)^2 - \hat{s}(1 + \hat{r}_A) - \hat{s}^2] \\
& \times (C_T + 2C_{TE}) - 6\sqrt{\hat{r}_A}\hat{s} \\
& \times (C_T - 2C_{TE})\} B_2^* f_T \\
& - 96\hat{m}_\ell \left[(1 + \sqrt{\hat{r}_A}) (1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s}) C_T \right. \\
& + 2(1 - \sqrt{\hat{r}_A}) (1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s}) C_{TE} \left. \right] A_1^* f_T \\
& - 96\hat{m}_\ell \left[(1 + \sqrt{\hat{r}_A}) (1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s}) C_T \right. \\
& - 2(1 - \sqrt{\hat{r}_A}) (1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s}) C_{TE} \left. \right] B_1^* f_T \\
& + 128 [(3v^2 - 1)(1 - \hat{r}_A)^2 \\
& + 6\hat{m}_\ell^2 (1 + 2\sqrt{\hat{r}_A} + \hat{r}_A + \hat{s}) - \hat{s}(1 + \hat{r}_A + \hat{s})] \\
& \times |C_T|^2 |f_T|^2 \\
& + 512 [(3v^2 - 1)(1 - \hat{r}_A)^2 \\
& + 6\hat{m}_\ell^2 (1 - 2\sqrt{\hat{r}_A} + \hat{r}_A + \hat{s}) - \hat{s}(1 + \hat{r}_A + \hat{s})] \\
& \times |C_{TE}|^2 |f_T|^2 \left. \right\},
\end{aligned}$$

$$\begin{aligned}
P_{LN} = & \frac{4\pi m_{A_b}^4 \sqrt{\lambda}}{\Delta \sqrt{\hat{s}}} \text{Im} \{ 4\hat{m}_\ell (1 - \hat{r}_A) (A_1^* D_1 + B_1^* E_1) \\
& + 32m_{A_b}^2 \hat{m}_\ell^2 (1 - \sqrt{\hat{r}_A}) (1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s}) \\
& \times (D_1^* + E_1^*) C_T f_T^S \\
& + 16\hat{m}_\ell \hat{s} (F_2 C_T^* f_T^* - 2H_2 C_{TE}^* f_T^*) \\
& + 4m_{A_b} \hat{m}_\ell \hat{s} (A_1^* E_3 - A_2^* E_1 + B_1^* D_3 - B_2^* D_1) \\
& + \hat{s} (1 + \sqrt{\hat{r}_A}) \{ (A_1^* + B_1^*) F_2 \\
& - 16m_{A_b} \hat{m}_\ell [F_2 + 2m_{A_b} \hat{m}_\ell (D_3 + E_3)] C_T^* f_T^{V*} \} \\
& + 32m_{A_b} \hat{m}_\ell^2 \hat{s} [(D_3 + E_3) C_T^* f_T^* - 2(D_3 - E_3) C_{TE}^* f_T^*] \\
& + \hat{s} (1 - \sqrt{\hat{r}_A}) (A_1 - B_1) H_2^* \\
& + \hat{s} \left[(1 - \sqrt{\hat{r}_A}) (D_1 + E_1) C_T^* f_T^* \right. \\
& + 2(1 + \sqrt{\hat{r}_A}) (D_1 - E_1) C_{TE}^* f_T^* \left. \right] \\
& + 4m_{A_b} \hat{m}_\ell \sqrt{\hat{r}_A} \hat{s} (A_1^* D_3 + A_2^* D_1 + B_1^* E_3 + B_2^* E_1) \\
& - 16m_{A_b}^2 \hat{m}_\ell \hat{s} (1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s}) \\
& \times \{ [F_2 + 2m_{A_b} \hat{m}_\ell (D_3 + E_3)] C_T^* f_T^{S*} \\
& - m_{A_b} \hat{s}^2 [B_2^* (F_2 - H_2 + 4m_{A_b} \hat{m}_\ell E_3) \\
& + A_2^* (F_2 + H_2 + 4m_{A_b} \hat{m}_\ell D_3) - v^2 D_2^* F_1] \\
& + \hat{s} v^2 \{ (D_1 + E_1) F_1^* - (D_1 - E_1) H_1^* \\
& - \sqrt{\hat{r}_A} [E_1^* (F_1 - H_1) + D_1^* (F_1 + H_1)] \}
\end{aligned}$$

$$\begin{aligned}
& + 8\hat{s} v^2 \{ (A_1 - B_1) C_T^* f_T^* + 2(A_1 + B_1) C_{TE}^* f_T^* \\
& - \sqrt{\hat{r}_A} [A_1^* (C_T - 2C_{TE}) f_T + B_1^* (C_T + 2C_{TE}) f_T] \} \\
& + 8m_{A_b} \hat{s} (1 - \hat{r}_A) v^2 [(A_2 - B_2 - D_2 - E_2) C_T^* f_T^* \\
& - 2(A_2 + B_2 - D_2 + E_2) C_{TE}^* f_T^*] \\
& - m_{A_b} \hat{s}^2 v^2 \\
& \times [8(A_1 - B_1) C_T^* f_T^{V*} - (F_1 - H_1) E_2^* + D_2 H_1^*] \\
& - 8m_{A_b}^2 \hat{s}^2 (1 - \sqrt{\hat{r}_A}) v^2 \\
& \times [(A_2 - B_2) C_T^* f_T^{V*} + 2(D_2 - E_2) C_{TE}^* f_T^{V*}] \\
& - 16m_{A_b} [2\hat{m}_\ell^2 (1 - \hat{r}_A) (D_1 + E_1) C_T^* f_T^{V*} \\
& + \hat{s}^2 v^2 (D_1 - E_1) C_{TE}^* f_T^{V*}] \}, \tag{20}
\end{aligned}$$

$$\begin{aligned}
P_{NL} = & - \frac{4\pi m_{A_b}^4 \sqrt{\lambda}}{\Delta \sqrt{\hat{s}}} \text{Im} \{ 4\hat{m}_\ell (1 - \hat{r}_A) (A_1^* D_1 + B_1^* E_1) \\
& + 32m_{A_b}^2 \hat{m}_\ell^2 (1 - \sqrt{\hat{r}_A}) (1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s}) \\
& \times (D_1^* + E_1^*) C_T f_T^S \\
& + 16\hat{m}_\ell \hat{s} (F_2 C_T^* f_T^* - 2H_2 C_{TE}^* f_T^*) \\
& + 4m_{A_b} \hat{m}_\ell \hat{s} (A_1^* E_3 - A_2^* E_1 + B_1^* D_3 - B_2^* D_1) \\
& + \hat{s} (1 + \sqrt{\hat{r}_A}) \{ (A_1^* + B_1^*) F_2 \\
& - 16m_{A_b} \hat{m}_\ell [F_2 + 2m_{A_b} \hat{m}_\ell (D_3 + E_3)] C_T^* f_T^{V*} \} \\
& + 32m_{A_b} \hat{m}_\ell^2 \hat{s} [(D_3 + E_3) C_T^* f_T^* - 2(D_3 - E_3) C_{TE}^* f_T^*] \\
& + \hat{s} (1 - \sqrt{\hat{r}_A}) (A_1 - B_1) H_2^* \\
& + \hat{s} \left[(1 - \sqrt{\hat{r}_A}) (D_1 + E_1) C_T^* f_T^* \right. \\
& + 2(1 + \sqrt{\hat{r}_A}) (D_1 - E_1) C_{TE}^* f_T^* \left. \right] \\
& + 4m_{A_b} \hat{m}_\ell \sqrt{\hat{r}_A} \hat{s} (A_1^* D_3 + A_2^* D_1 + B_1^* E_3 + B_2^* E_1) \\
& - 4m_{A_b} \hat{m}_\ell \sqrt{\hat{r}_A} \hat{s} (A_1^* D_3 + A_2^* D_1 + B_1^* E_3 + B_2^* E_1) \\
& - m_{A_b} \hat{s}^2 [B_2^* (F_2 - H_2 + 4m_{A_b} \hat{m}_\ell E_3) \\
& + A_2^* (F_2 + H_2 + 4m_{A_b} \hat{m}_\ell D_3) - v^2 D_2^* F_1] \\
& - \hat{s} v^2 \{ (D_1 + E_1) F_1^* - (D_1 - E_1) H_1^* \\
& - \sqrt{\hat{r}_A} [E_1^* (F_1 - H_1) + D_1^* (F_1 + H_1)] \} \\
& - 8\hat{s} v^2 \{ (A_1 - B_1) C_T^* f_T^* + 2(A_1 + B_1) C_{TE}^* f_T^* \\
& - \sqrt{\hat{r}_A} [A_1^* (C_T - 2C_{TE}) f_T + B_1^* (C_T + 2C_{TE}) f_T] \} \\
& - 8m_{A_b} \hat{s} (1 - \hat{r}_A) v^2 \\
& \times [(A_2 - B_2 + D_2 + E_2) C_T^* f_T^* \\
& - 2(A_2 + B_2 + D_2 - E_2) C_{TE}^* f_T^*] \\
& + m_{A_b} \hat{s}^2 v^2 \\
& \times [8(A_1 - B_1) C_T^* f_T^{V*} - (F_1 - H_1) E_2^* + D_2 H_1^*]
\end{aligned}$$

$$\begin{aligned}
& +8m_{A_b}^2 \hat{s}^2 \left(1 - \sqrt{\hat{r}_A}\right) v^2 \\
& \times \left[(A_2 - B_2) C_T^* f_T^{V*} - 2(D_2 - E_2) C_{TE}^* f_T^{V*} \right] \\
& -16m_{A_b} \left[2\hat{m}_\ell^2 (1 - \hat{r}_A) (D_1 + E_1) C_T^* f_T^{V*} \right. \\
& \left. + \hat{s}^2 v^2 (D_1 - E_1) C_{TE}^* f_T^{V*} \right] \}, \quad (21)
\end{aligned}$$

$$\begin{aligned}
P_{LT} &= \frac{4\pi m_{A_b}^4 \sqrt{\lambda} v}{\Delta \sqrt{\hat{s}}} \operatorname{Re} \{ 4\hat{m}_\ell (1 - \hat{r}_A) \\
& \times \left[|D_1|^2 + |E_1|^2 - 32 \left(|C_T|^2 + 4|C_{TE}|^2 \right) |f_T|^2 \right] \\
& -4\hat{m}_\ell \hat{s} (A_1 D_1^* - B_1 E_1^* + 4F_1^* C_T f_T) \\
& +4\hat{m}_\ell \hat{s} \{ 8H_1 C_{TE}^* f_T^* - m_{A_b} [B_1 D_2^* + (A_2 + D_2 - D_3) E_1^* \\
& - A_1 E_2^* - (B_2 - E_2 + E_3) D_1^*] \} \\
& +\hat{s} \left(1 - \sqrt{\hat{r}_A} \right) \left[(A_1 - B_1) H_1^* - (D_1 - E_1) H_2^* \right. \\
& \left. + 128m_{A_b} \hat{m}_\ell \left(4|C_{TE}|^2 + |C_T|^2 \right) f_T f_T^{V*} \right] \\
& -\hat{s} \left(1 + \sqrt{\hat{r}_A} \right) \\
& \times \left[(A_1 + B_1) F_1^* - (D_1 + E_1) F_2^* - 16m_{A_b} \hat{m}_\ell F_1 C_T^* f_T^{V*} \right] \\
& -8\hat{s} \left\{ \left[(A_1^* + D_1^*) - \sqrt{\hat{r}_A} (B_1^* + E_1^*) \right] (C_T + 2C_{TE}) f_T \right. \\
& \left. + \left[(B_1^* - E_1^*) - \sqrt{\hat{r}_A} (A_1^* - D_1^*) \right] (C_T - 2C_{TE}) f_T \right\} \\
& +4m_{A_b} \hat{s} (1 - \hat{r}_A) \\
& \times \{ 2[A_2^* (C_T - 2C_{TE}) f_T + B_2^* (C_T + 2C_{TE}) f_T \\
& - D_2^* (C_T - 2C_{TE}) f_T + E_2^* (C_T + 2C_{TE}) f_T] \\
& + m_{A_b} \hat{m}_\ell (A_2 D_2^* - B_2 E_2^*) \} \\
& +4m_{A_b} \hat{m}_\ell \sqrt{\hat{r}_A} \hat{s} [A_1 D_2^* + (A_2 + D_2 + D_3) D_1^* \\
& - B_1 E_2^* - (B_2 - E_2 - E_3) E_1^*] \\
& +m_{A_b} \hat{s}^2 \{ 8[2(A_1 - B_1) C_{TE}^* + (D_1 - E_1) C_T^*] f_T^{V*} \\
& + (A_2 - B_2) H_1^* + (A_2 + B_2) F_1^* - (D_2 - E_2) H_2^* \\
& - (D_2 + E_2) F_2^* - 4m_{A_b} \hat{m}_\ell (D_2 D_3^* + E_2 E_3^*) \} \\
& +8m_{A_b}^2 \hat{s}^2 \left(1 - \sqrt{\hat{r}_A} \right) \\
& \times [2(A_2 - B_2) C_{TE}^* + (D_2 - E_2) C_T^*] f_T^{V*} \\
& \left. + 16m_{A_b}^2 \hat{m}_\ell \hat{s} \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) F_1^* C_T f_T^S \right\}, \quad (22)
\end{aligned}$$

$$\begin{aligned}
P_{TL} &= \frac{4\pi m_{A_b}^4 \sqrt{\lambda} v}{\Delta \sqrt{\hat{s}}} \operatorname{Re} \{ 4\hat{m}_\ell (1 - \hat{r}_A) \\
& \times \left[|D_1|^2 + |E_1|^2 - 32 \left(|C_T|^2 + 4|C_{TE}|^2 \right) |f_T|^2 \right] \\
& +4\hat{m}_\ell \hat{s} (A_1 D_1^* - B_1 E_1^* + 4F_1^* C_T f_T) \\
& -4\hat{m}_\ell \hat{s} \{ 8H_1 C_{TE}^* f_T^* - m_{A_b} [B_1 D_2^* + (A_2 - D_2 + D_3) E_1^* \\
& - A_1 E_2^* - (B_2 + E_2 - E_3) D_1^*] \} \\
& -\hat{s} \left(1 - \sqrt{\hat{r}_A} \right) \left[(A_1 - B_1) H_1^* + (D_1 - E_1) H_2^* \right.
\end{aligned}$$

$$\begin{aligned}
& \left. - 128m_{A_b} \hat{m}_\ell \left(4|C_{TE}|^2 + |C_T|^2 \right) f_T f_T^{V*} \right] \\
& +\hat{s} \left(1 + \sqrt{\hat{r}_A} \right) \\
& \times \left[(A_1 + B_1) F_1^* + (D_1 + E_1) F_2^* - 16m_{A_b} \hat{m}_\ell F_1 C_T^* f_T^{V*} \right] \\
& -8\hat{s} \left\{ \left[(A_1^* - D_1^*) - \sqrt{\hat{r}_A} (B_1^* - E_1^*) \right] (C_T + 2C_{TE}) f_T \right. \\
& \left. + \left[(B_1^* + E_1^*) - \sqrt{\hat{r}_A} (A_1^* + D_1^*) \right] (C_T - 2C_{TE}) f_T \right\} \\
& +4m_{A_b} \hat{s} (1 - \hat{r}_A) \\
& \times \{ 2[A_2^* (C_T - 2C_{TE}) f_T + B_2^* (C_T + 2C_{TE}) f_T \\
& + D_2^* (C_T - 2C_{TE}) f_T - E_2^* (C_T + 2C_{TE}) f_T] \\
& - m_{A_b} \hat{m}_\ell (A_2 D_2^* - B_2 E_2^*) \} \\
& -4m_{A_b} \hat{m}_\ell \sqrt{\hat{r}_A} \hat{s} \\
& \times [A_1 D_2^* + (A_2 - D_2 - D_3) D_1^* - B_1 E_2^* \\
& - (B_2 + E_2 + E_3) E_1^*] \\
& +m_{A_b} \hat{s}^2 \{ 8[2(A_1 - B_1) C_{TE}^* - (D_1 - E_1) C_T^*] f_T^{V*} \\
& - (A_2 - B_2) H_1^* - (A_2 + B_2) F_1^* - (D_2 - E_2) H_2^* \\
& - (D_2 + E_2) F_2^* - 4m_{A_b} \hat{m}_\ell (D_2 D_3^* + E_2 E_3^*) \} \\
& +8m_{A_b}^2 \hat{s}^2 \left(1 - \sqrt{\hat{r}_A} \right) \\
& \times [2(A_2 - B_2) C_{TE}^* - (D_2 - E_2) C_T^*] f_T^{V*} \\
& \left. - 16m_{A_b}^2 \hat{m}_\ell \hat{s} \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) F_1^* C_T f_T^S \right\}, \quad (23)
\end{aligned}$$

$$\begin{aligned}
P_{NT} &= \frac{16m_{A_b}^4 v}{3\Delta} \operatorname{Im} \{ 4\lambda (A_1 D_1^* + B_1 E_1^*) \\
& +32m_{A_b} \hat{m}_\ell \lambda [(D_1 + E_1) C_T^* f_T^{V*} - (D_2 + E_2) C_T^* f_T^* \\
& + 2(D_2 - E_2) C_{TE}^* f_T^*] \\
& +32m_{A_b}^2 \hat{m}_\ell \lambda \left(1 + \sqrt{\hat{r}_A} \right) \\
& \times [(A_1 + B_1) C_{TE}^* f_T^{S*} + (D_1 + E_1) C_T^* f_T^{S*}] \\
& +6\hat{m}_\ell \left(1 + \sqrt{\hat{r}_A} \right) \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) (D_1 - E_1) H_1^* \\
& -6\hat{m}_\ell \left(1 - \sqrt{\hat{r}_A} \right) \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) (D_1 + E_1) F_1^* \\
& -32m_{A_b}^3 \hat{m}_\ell \lambda \hat{s} [(A_2 + B_2) C_{TE}^* + (D_2 + E_2) C_T^*] f_T^{S*} \\
& +4m_{A_b}^2 \lambda \hat{s} (A_2 D_2 + B_2 E_2) \\
& +256m_{A_b}^2 \lambda \hat{s} \\
& \times \left\{ \operatorname{Re} [f_T f_T^{S*}] - m_{A_b} \left(1 + \sqrt{\hat{r}_A} \right) \operatorname{Re} [f_T^S f_T^{V*}] \right\} C_T C_{TE}^* \\
& +3\lambda \hat{s} \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \\
& \times [H_1 H_2^* - 2m_{A_b} \hat{m}_\ell (D_3 - E_3) H_1^*] \\
& +96m_{A_b} \hat{s} \left(1 + \sqrt{\hat{r}_A} \right) \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \\
& \times \{ 8\operatorname{Re} [f_T f_T^{V*}] C_T C_{TE}^* - m_{A_b} \hat{m}_\ell (A_2 + B_2) C_{TE}^* f_T^{V*} \} \\
& -\hat{s} \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \left\{ 128\lambda m_{A_b}^4 |f_T^S|^2 C_T C_{TE}^* \right.
\end{aligned}$$

$$\begin{aligned}
& + 3 [F_2 + 2m_{A_b} \hat{m}_\ell (D_3 + E_3)] F_1^* \} \\
& - 32m_{A_b} \left[(1 - \hat{r}_\Lambda)^2 + \hat{s} \left(1 - 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) - 2\hat{s}^2 \right] \\
& \times \left[4m_{A_b} \hat{s} |f_T^V|^2 C_T C_{TE}^* - \hat{m}_\ell (A_1 + B_1) C_{TE}^* f_T^{V*} \right] \\
& + 1536\sqrt{\hat{r}_\Lambda} \hat{s} |f_T|^2 C_T C_{TE}^* \\
& - 48\hat{m}_\ell \left\{ (1 - \hat{r}_\Lambda) \left[C_T - 2C_{TE} + \sqrt{\hat{r}_\Lambda} (C_T + 2C_{TE}) \right] \right. \\
& \left. - \hat{s} \left[C_T - 2C_{TE} - \sqrt{\hat{r}_\Lambda} (C_T + 2C_{TE}) \right] \right\} B_1^* f_T \\
& + 16m_{A_b} \hat{m}_\ell \left\{ 2(1 - \hat{r}_\Lambda)^2 (C_T - 2C_{TE}) \right. \\
& \left. - \hat{s} \left[\left(1 - 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) C_T - 2 \left(1 + 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) C_{TE} \right] \right. \\
& \left. - \hat{s}^2 (C_T - 2C_{TE}) \right\} A_2^* f_T \\
& - 16m_{A_b} \hat{m}_\ell \left\{ 2(1 - \hat{r}_\Lambda)^2 (C_T + 2C_{TE}) \right. \\
& \left. - \hat{s} \left[\left(1 - 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) C_T + 2 \left(1 + 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) C_{TE} \right] \right. \\
& \left. - \hat{s}^2 (C_T + 2C_{TE}) \right\} B_2^* f_T \\
& + 48\hat{m}_\ell \left\{ (1 - \hat{r}_\Lambda) \left[C_T + 2C_{TE} + \sqrt{\hat{r}_\Lambda} (C_T - 2C_{TE}) \right] \right. \\
& \left. - \hat{s} \left[C_T + 2C_{TE} - \sqrt{\hat{r}_\Lambda} (C_T - 2C_{TE}) \right] \right\} A_1^* f_T \} , \quad (24)
\end{aligned}$$

$$\begin{aligned}
P_{TN} = & -\frac{16m_{A_b}^4 v}{3\Delta} \text{Im} \{ 4\lambda (A_1 D_1^* + B_1 E_1^*) \\
& + 32m_{A_b} \hat{m}_\ell \lambda \left[(D_1 + E_1) C_T^* f_T^{V*} - (D_2 + E_2) C_T^* f_T^* \right. \\
& \left. + 2(D_2 - E_2) C_{TE}^* f_T^* \right] \\
& - 32m_{A_b}^2 \hat{m}_\ell \lambda \left(1 + \sqrt{\hat{r}_\Lambda} \right) \\
& \times \left[(A_1 + B_1) C_{TE}^* f_T^{S*} - (D_1 + E_1) C_T^* f_T^{S*} \right] \\
& - 6\hat{m}_\ell \left(1 + \sqrt{\hat{r}_\Lambda} \right) \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) (D_1 - E_1) H_1^* \\
& + 6\hat{m}_\ell \left(1 - \sqrt{\hat{r}_\Lambda} \right) \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) (D_1 + E_1) F_1^* \\
& + 32m_{A_b}^3 \hat{m}_\ell \lambda \hat{s} \left[(A_2 + B_2) C_{TE}^* - (D_2 + E_2) C_T^* \right] f_T^{S*} \\
& + 4m_{A_b}^2 \lambda \hat{s} (A_2^* D_2 + B_2^* E_2) \\
& - 256m_{A_b}^2 \lambda \hat{s} \\
& \times \left\{ \text{Re} [f_T f_T^{S*}] - m_{A_b} \left(1 + \sqrt{\hat{r}_\Lambda} \right) \text{Re} [f_T^S f_T^{V*}] \right\} C_T C_{TE}^* \\
& - 3\lambda \hat{s} \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times [H_1 H_2^* - 2m_{A_b} \hat{m}_\ell (D_3 - E_3) H_1^*] \\
& - 96m_{A_b} \hat{s} \left(1 + \sqrt{\hat{r}_\Lambda} \right) \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times \left\{ 8\text{Re} [f_T f_T^{V*}] C_T C_{TE}^* - m_{A_b} \hat{m}_\ell (A_2 + B_2) C_{TE}^* f_T^{V*} \right\} \\
& + \hat{s} \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \left\{ 128\lambda m_{A_b}^4 |f_T^S|^2 C_T C_{TE}^* \right. \\
& \left. + 3 [F_2 + 2m_{A_b} \hat{m}_\ell (D_3 + E_3)] F_1^* \right\} \\
& + 32m_{A_b} \left[(1 - \hat{r}_\Lambda)^2 + \hat{s} \left(1 - 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) - 2\hat{s}^2 \right]
\end{aligned}$$

$$\begin{aligned}
& \times \left[4m_{A_b} \hat{s} |f_T^V|^2 C_T C_{TE}^* - \hat{m}_\ell (A_1 + B_1) C_{TE}^* f_T^{V*} \right] \\
& - 1536\sqrt{\hat{r}_\Lambda} \hat{s} |f_T|^2 C_T C_{TE}^* \\
& + 48\hat{m}_\ell \left\{ (1 - \hat{r}_\Lambda) \left[C_T - 2C_{TE} + \sqrt{\hat{r}_\Lambda} (C_T + 2C_{TE}) \right] \right. \\
& \left. - \hat{s} \left[C_T - 2C_{TE} - \sqrt{\hat{r}_\Lambda} (C_T + 2C_{TE}) \right] \right\} B_1^* f_T \\
& - 16m_{A_b} \hat{m}_\ell \left\{ 2(1 - \hat{r}_\Lambda)^2 (C_T - 2C_{TE}) \right. \\
& \left. - \hat{s} \left[\left(1 - 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) C_T - 2 \left(1 + 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) C_{TE} \right] \right. \\
& \left. - \hat{s}^2 (C_T - 2C_{TE}) \right\} A_2^* f_T \\
& + 16m_{A_b} \hat{m}_\ell \left\{ 2(1 - \hat{r}_\Lambda)^2 (C_T + 2C_{TE}) \right. \\
& \left. - \hat{s} \left[\left(1 - 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) C_T + 2 \left(1 + 6\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda \right) C_{TE} \right] \right. \\
& \left. - \hat{s}^2 (C_T + 2C_{TE}) \right\} B_2^* f_T \\
& - 48\hat{m}_\ell \left\{ (1 - \hat{r}_\Lambda) \left[C_T + 2C_{TE} + \sqrt{\hat{r}_\Lambda} (C_T - 2C_{TE}) \right] \right. \\
& \left. - \hat{s} \left[C_T + 2C_{TE} - \sqrt{\hat{r}_\Lambda} (C_T - 2C_{TE}) \right] \right\} A_1^* f_T \} , \quad (25)
\end{aligned}$$

$$\begin{aligned}
P_{NN} = & \frac{8m_{A_b}^4}{3\hat{s}\Delta} \text{Re} \left\{ 96m_{A_b}^2 \hat{m}_\ell \lambda \hat{s} \left(1 + \sqrt{\hat{r}_\Lambda} \right) (A_1 + B_1) C_T^* f_T^{S*} \right. \\
& + 96\hat{m}_\ell^2 \sqrt{\hat{r}_\Lambda} \hat{s} (A_1 B_1^* + D_1 E_1^*) \\
& - 48m_{A_b} \hat{m}_\ell^2 \sqrt{\hat{r}_\Lambda} \hat{s} (1 - \hat{r}_\Lambda + \hat{s}) (A_1 A_2^* + B_1 B_2^*) \\
& + 96m_{A_b} \hat{m}_\ell \hat{s} \left(1 + \sqrt{\hat{r}_\Lambda} \right)^2 \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times (A_1 + B_1) C_T^* f_T^{V*} \\
& + 12\hat{m}_\ell \hat{s} \left(1 - \sqrt{\hat{r}_\Lambda} \right) \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times (D_1 + E_1) F_2^* \\
& - 96m_{A_b}^3 \hat{m}_\ell \lambda \hat{s}^2 (A_2 + B_2) C_T^* f_T^{S*} \\
& - 12\hat{m}_\ell \hat{s} \left(1 + \sqrt{\hat{r}_\Lambda} \right) \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times \left[(D_1 - E_1) H_2^* + 8m_{A_b}^2 \hat{s} (A_2 + B_2) C_T^* f_T^{V*} \right] \\
& + 3\hat{s}^2 \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times \left[|F_2|^2 + 4m_{A_b} \hat{m}_\ell (D_3 + E_3) F_2^* \right] \\
& + 24m_{A_b} \hat{m}_\ell^2 \hat{s} \left[m_{A_b} \hat{s} (1 + \hat{r}_\Lambda - \hat{s}) \left(|D_3|^2 + |E_3|^2 \right) \right. \\
& \left. + 2\sqrt{\hat{r}_\Lambda} (1 - \hat{r}_\Lambda + \hat{s}) (D_1 D_3^* + E_1 E_3^*) \right] \\
& + 48m_{A_b} \hat{m}_\ell^2 \hat{s} (1 - \hat{r}_\Lambda - \hat{s}) \\
& \times (A_1 B_2^* + A_2 B_1^* + D_1 E_3^* + D_3 E_1^*) \\
& - 4 \left[\lambda \hat{s} + 2\hat{m}_\ell^2 (1 + \hat{r}_\Lambda^2 - 2\hat{r}_\Lambda + \hat{r}_\Lambda \hat{s} + \hat{s} - 2\hat{s}^2) \right] \\
& \times \left(|A_1|^2 + |B_1|^2 - |D_1|^2 - |E_1|^2 \right) \\
& + 3\hat{s}^2 \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times \left[4m_{A_b} \hat{m}_\ell (D_3 - E_3) H_2^* + |H_2|^2 \right]
\end{aligned}$$

$$\begin{aligned}
& +96m_{A_b}^2 \hat{m}_\ell^2 \sqrt{\hat{r}_A} \hat{s}^2 (A_2 B_2^* + D_3 E_3^*) \\
& +64m_{A_b}^2 \lambda \hat{s}^2 \left[(3-2v^2) |C_T|^2 - 4v^2 |C_{TE}|^2 \right] \\
& \times \left\{ 2f_T f_T^{S*} - m_{A_b} \left[2 \left(1 + \sqrt{\hat{r}_A} \right) f_T^S f_T^{V*} \right. \right. \\
& \left. \left. + m_{A_b} \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) |f_T^S|^2 \right] \right\} \\
& -4m_{A_b}^2 \lambda \hat{s}^2 v^2 \left(|D_2|^2 + |E_2|^2 \right) \\
& +4m_{A_b}^2 \hat{s} \{ \lambda \hat{s} - 2\hat{m}_\ell^2 [2(1 + \hat{r}_A^2) \\
& - \hat{s}(1 + \hat{s}) - \hat{r}_A(4 + \hat{s})] \} \left(|A_2|^2 + |B_2|^2 \right) \\
& +96m_{A_b} \hat{m}_\ell \hat{s}^2 \left[\left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) B_2^* f_T C_T \right. \\
& \left. + 2 \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) B_2^* f_T C_{TE} \right] \\
& -96\hat{m}_\ell \hat{s} \left[\left(1 + \sqrt{\hat{r}_A} \right) \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) A_1^* f_T C_T \right. \\
& \left. + 2 \left(1 - \sqrt{\hat{r}_A} \right) \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) A_1^* f_T C_{TE} \right] \\
& -96\hat{m}_\ell \hat{s} \left[\left(1 + \sqrt{\hat{r}_A} \right) \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) B_1^* f_T C_T \right. \\
& \left. - 2 \left(1 - \sqrt{\hat{r}_A} \right) \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) B_1^* f_T C_{TE} \right] \\
& -768\hat{s} \left\{ \left[\hat{m}_\ell^2 \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) - \sqrt{\hat{r}_A} \hat{s} \right] |C_T|^2 \right. \\
& \left. + 4 \left[\hat{m}_\ell^2 \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) + \sqrt{\hat{r}_A} \hat{s} \right] |C_{TE}|^2 \right\} |f_T|^2 \\
& +96m_{A_b} \hat{m}_\ell \hat{s}^2 \left[\left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) C_T^* \right. \\
& \left. - 2 \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) C_{TE}^* \right] A_2 f_T^* \\
& -3\hat{s}^2 v^2 \left[\left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) |F_1|^2 \right. \\
& \left. + \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) |H_1|^2 \right] \\
& +384m_{A_b} \hat{s}^2 \left(1 + \sqrt{\hat{r}_A} \right) \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \\
& \times \left(|C_T|^2 - 4v^2 |C_{TE}|^2 \right) f_T f_T^{V*} \\
& -64m_{A_b}^2 \hat{s} \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \\
& \times \left\{ \left[8\hat{m}_\ell^2 \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \right. \right. \\
& \left. \left. + \hat{s} \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A + 2\hat{s} \right) \right] |C_T|^2 \right. \\
& \left. - 4\hat{s} v^2 \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A + 2\hat{s} \right) |C_{TE}|^2 \right\} |f_T^V|^2 \left. \right\}, \quad (26) \\
P_{TT} = & \frac{8m_{A_b}^4}{3\hat{s}\Delta} \text{Re} \left\{ 32m_{A_b}^2 \hat{m}_\ell \lambda \hat{s} \left(1 + \sqrt{\hat{r}_A} \right) \left(A_1 + B_1 \right) C_T^* f_T^{S*} \right. \\
& -96\hat{m}_\ell^2 \sqrt{\hat{r}_A} \hat{s} \left(A_1 B_1^* + D_1 E_1^* \right) \\
& -48m_{A_b} \hat{m}_\ell^2 \sqrt{\hat{r}_A} \hat{s} \left(1 - \hat{r}_A + \hat{s} \right) \left(D_1 D_3^* + E_1 E_3^* \right) \\
& \left. -12\hat{m}_\ell \hat{s} \left(1 - \sqrt{\hat{r}_A} \right) \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \times \left(D_1 + E_1 \right) F_2^* \\
& -32m_{A_b}^3 \hat{m}_\ell \lambda \hat{s}^2 \left(A_2 + B_2 \right) C_T^* f_T^{S*} \\
& -96m_{A_b}^2 \hat{m}_\ell^2 \sqrt{\hat{r}_A} \hat{s}^2 \left(A_2 B_2^* + D_3 E_3^* \right) \\
& +12\hat{m}_\ell \hat{s} \left(1 + \sqrt{\hat{r}_A} \right) \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \\
& \times \left[\left(D_1 - E_1 \right) H_2^* + 8m_{A_b}^2 \hat{s} \left(A_2 + B_2 \right) C_T^* f_T^{V*} \right] \\
& -3\hat{s}^2 \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \\
& \times \left[|F_2|^2 + 4m_{A_b} \hat{m}_\ell \left(D_3 + E_3 \right) F_2^* \right] \\
& -24m_{A_b} \hat{m}_\ell^2 \hat{s} \left[m_{A_b} \hat{s} \left(1 + \hat{r}_A - \hat{s} \right) \left(|D_3|^2 + |E_3|^2 \right) \right. \\
& \left. - 2\sqrt{\hat{r}_A} \left(1 - \hat{r}_A + \hat{s} \right) \left(A_1 A_2^* + B_1 B_2^* \right) \right] \\
& -48m_{A_b} \hat{m}_\ell^2 \hat{s} \left(1 - \hat{r}_A - \hat{s} \right) \\
& \times \left(A_1 B_2^* + A_2 B_1^* + D_1 E_3^* + D_3 E_1^* \right) \\
& -4 \left[\lambda \hat{s} - 2\hat{m}_\ell^2 \left(1 + \hat{r}_A^2 - 2\hat{r}_A + \hat{r}_A \hat{s} + \hat{s} - 2\hat{s}^2 \right) \right] \\
& \times \left(|A_1|^2 + |B_1|^2 \right) \\
& +4m_{A_b}^2 \hat{s} \left\{ \lambda \hat{s} + \hat{m}_\ell^2 \left[4 \left(1 - \hat{r}_A \right)^2 - 2\hat{s} \left(1 + \hat{r}_A \right) - 2\hat{s}^2 \right] \right. \\
& \left. \times \left(|A_2|^2 + |B_2|^2 \right) \right. \\
& \left. + 4 \left\{ \lambda \hat{s} - 2\hat{m}_\ell^2 \left[5 \left(1 - \hat{r}_A \right)^2 - 7\hat{s} \left(1 + \hat{r}_A \right) + 2\hat{s}^2 \right] \right\} \right. \\
& \left. \times \left(|D_1|^2 + |E_1|^2 \right) \right. \\
& \left. + 32m_{A_b} \hat{m}_\ell \hat{s} \right. \\
& \left. \times \left[\left(1 - \hat{r}_A \right)^2 - \hat{s} \left(5 - 6\sqrt{\hat{r}_A} + 5\hat{r}_A \right) + 4\hat{s}^2 \right] \right. \\
& \left. \times \left[\left(A_1 + B_1 \right) C_T f_T^{V*} \right] \right. \\
& \left. - 3\hat{s}^2 \left(1 - 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) \right. \\
& \left. \times \left[4m_{A_b} \hat{m}_\ell \left(D_3 - E_3 \right) H_2^* + |H_2|^2 \right] \right. \\
& \left. - 64m_{A_b}^2 \lambda \hat{s}^2 \left[\left(2v^2 - 1 \right) |C_T|^2 - 4v^2 |C_{TE}|^2 \right] \right. \\
& \left. \times \left\{ 2f_T f_T^{S*} - m_{A_b} \left[2 \left(1 + \sqrt{\hat{r}_A} \right) f_T^S f_T^{V*} \right. \right. \right. \\
& \left. \left. + m_{A_b} \left(1 + 2\sqrt{\hat{r}_A} + \hat{r}_A - \hat{s} \right) |f_T^S|^2 \right] \right\} \\
& -32m_{A_b} \hat{m}_\ell \hat{s} \\
& \times \left\{ \left[4 \left(1 - \hat{r}_A \right)^2 - 5\hat{s} \left(1 + \hat{r}_A \right) + \hat{s}^2 \right] \right. \\
& \left. \times A_2^* \left(C_T - 2C_{TE} \right) f_T \right. \\
& \left. - 6\sqrt{\hat{r}_A} \hat{s} A_2^* \left(C_T + 2C_{TE} \right) f_T \right\} \\
& -32m_{A_b} \hat{m}_\ell \hat{s} \\
& \times \left\{ \left[4 \left(1 - \hat{r}_A \right)^2 - 5\hat{s} \left(1 + \hat{r}_A \right) + \hat{s}^2 \right] \right. \\
& \left. \times B_2^* \left(C_T + 2C_{TE} \right) f_T \right. \\
& \left. - 6\sqrt{\hat{r}_A} \hat{s} B_2^* \left(C_T - 2C_{TE} \right) f_T \right\}
\end{aligned}$$

$$\begin{aligned}
& +256 \left\{ -3\sqrt{\hat{r}_\Lambda} \hat{s}^2 \right. \\
& + \hat{m}_\ell^2 \left[4(1 - \hat{r}_\Lambda)^2 - \hat{s} \left(5 - 6\sqrt{\hat{r}_\Lambda} + 5\hat{r}_\Lambda \right) + \hat{s}^2 \right] \Big\} \\
& \times |C_T|^2 |f_T|^2 \\
& + 1024 \left\{ 3\sqrt{\hat{r}_\Lambda} \hat{s}^2 \right. \\
& + \hat{m}_\ell^2 \left[4(1 - \hat{r}_\Lambda)^2 - \hat{s} \left(5 + 6\sqrt{\hat{r}_\Lambda} + 5\hat{r}_\Lambda \right) + \hat{s}^2 \right] \Big\} \\
& \times |C_{TE}|^2 |f_T|^2 \\
& + 96\hat{m}_\ell \hat{s} \left[\left(1 + \sqrt{\hat{r}_\Lambda} \right) \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) C_T^* \right. \\
& + 2 \left(1 - \sqrt{\hat{r}_\Lambda} \right) \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) C_{TE}^* \Big] A_1 f_T^* \\
& + 96\hat{m}_\ell \hat{s} \left[\left(1 + \sqrt{\hat{r}_\Lambda} \right) \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) C_T^* \right. \\
& - 2 \left(1 - \sqrt{\hat{r}_\Lambda} \right) \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) C_{TE}^* \Big] B_1 f_T^* \\
& - 4m_{\Lambda_b}^2 \lambda \hat{s}^2 v^2 \left(|D_2|^2 + |E_2|^2 \right) \\
& + 3\hat{s}^2 v^2 \left[\left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) |F_1|^2 \right. \\
& + \left. \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) |H_1|^2 \right] \\
& - 384m_{\Lambda_b} \hat{s}^2 \left(1 + \sqrt{\hat{r}_\Lambda} \right) \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times \left(|C_T|^2 - 4v^2 |C_{TE}|^2 \right) f_T f_T^{V*} \\
& - 64m_{\Lambda_b}^2 \hat{s} \left(1 - 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \\
& \times \left\{ \left[8\hat{m}_\ell^2 \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda - \hat{s} \right) \right. \right. \\
& - \left. \left. \hat{s} \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda + 2\hat{s} \right) \right] |C_T|^2 \right. \\
& + \left. 4\hat{s} v^2 \left(1 + 2\sqrt{\hat{r}_\Lambda} + \hat{r}_\Lambda + 2\hat{s} \right) |C_{TE}|^2 \right\} |f_T^V|^2 \Big\}. \quad (27)
\end{aligned}$$

4 Numerical analysis

We start this section by presenting the numerical results for all possible double-lepton polarization asymmetries. The values of the input parameters we use in our calculations are $|V_{tb}V_{ts}^*| = 0.0385$, $m_\tau = 1.77$ GeV, $m_\mu = 0.106$ GeV, $m_b = 4.8$ GeV. For the Wilson coefficients we use their SM values which are given by $C_7^{\text{SM}} = -0.313$, $C_9^{\text{SM}} = 4.344$ and $C_{10}^{\text{SM}} = -4.669$. The magnitude of C_7^{eff} is quite well constrained from $b \rightarrow s\gamma$ decay, and hence well established. Moreover, we will fix the values of the Wilson coefficients, i.e., C_{BR} and C_{SL} are both related to C_7^{eff} as follows: $C_{BR} = -2m_b C_7^{\text{eff}}$ and $C_{SL} = -2m_s C_7^{\text{eff}}$. As far as the Wilson coefficient C_9^{SM} is considered, we take into account only the short distance contributions, and we neglect the long distance contributions, coming from the production of $\bar{c}c$ pair at intermediate states. It is well known that the form factors are the main and the most important input parameters necessary in the numerical calculations. The

Table 1. Form factors for $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in a three parameter fit

	$F(0)$	a_F	b_F
F_1	0.462	-0.0182	-0.000176
F_2	-0.077	-0.0685	0.00146

calculation of the form factors of $\Lambda_b \rightarrow \Lambda$ transition does not exist at present. But we can use the results from QCD sum rules in corporation with HQET [22, 23]. We noted earlier that HQET allows us to establish relations among the form factors and reduces the number of independent form factors into two. In [22, 23], the q^2 dependence of these form factors are given as follows:

$$F(\hat{s}) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2}.$$

The values of the parameters $F(0)$, a_F and b_F are given in Table 1.

In the further numerical analysis, the values of the new Wilson coefficients which describe new physics beyond the SM are needed. In the numerical calculations we will vary all new Wilson coefficients in the range $-|C_{10}^{\text{SM}}| \leq C_X \leq |C_{10}^{\text{SM}}|$. The experimental results on the branching ratio of the $B \rightarrow K^* \ell^+ \ell^-$ decay [13, 14] and the bound on the branching ratio of $B \rightarrow \mu^+ \mu^-$ [24] suggest that this is the right order of magnitude for the vector and scalar interaction coefficients. Here, we emphasize that the existing experimental results on the $B \rightarrow K^* \ell^+ \ell^-$ and $B \rightarrow K \ell^+ \ell^-$ decays put stronger restrictions on some of the new Wilson coefficients. For example, $-2 \leq C_{LL} \leq 0$, $0 \leq C_{RL} \leq 2.3$, $-1.5 \leq C_T \leq 1.5$ and $-3.3 \leq C_{TE} \leq 2.6$, and all of the remaining Wilson coefficients vary in the region $-|C_{10}^{\text{SM}}| \leq C_X \leq |C_{10}^{\text{SM}}|$.

It follows from the explicit expressions of the double-lepton polarization asymmetries that they depend on q^2 and the new Wilson coefficients. For this reason there may appear difficulties in studying the dependencies of the physical properties on both parameters at the same time. Hence, it is necessary to eliminate the dependence of P_{ij} on one of these parameters. In the present work, we eliminate the q^2 dependence of P_{ij} by performing the integration over q^2 in the kinematically allowed region. The averaging of P_{ij} over q^2 is defined as

$$\langle P_{ij} \rangle = \frac{\int_{4m_\ell^2}^{(m_{\Lambda_b} - m_\Lambda)^2} P_{ij} \frac{d\mathcal{B}}{dq^2} dq^2}{\int_{4m_\ell^2}^{(m_{\Lambda_b} - m_\Lambda)^2} \frac{d\mathcal{B}}{dq^2} dq^2}. \quad (28)$$

In Figs. 1–5 we present the correlation of the averaged double-lepton polarization asymmetries on the branching ratio for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay. From these figures we obtain the following results.

- (1) There exist regions of the new Wilson coefficients where $\langle P_{LL} \rangle$ departs considerably from the SM result when $\mathcal{B}(\Lambda_b \rightarrow \Lambda \mu^+ \mu^-)$ is close to the SM prediction.
- (2) $\langle P_{LT} \rangle$ and $\langle P_{TL} \rangle$ are both very sensitive to the existence of the tensor and scalar interactions. We observe that

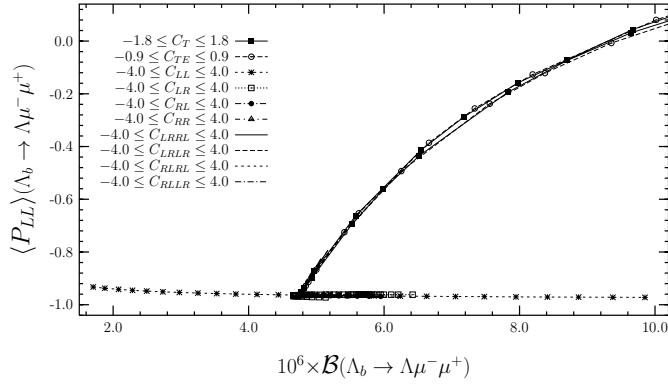


Fig. 1. Parametric plot of the correlation between the averaged double-lepton polarization asymmetry $\langle P_{LL} \rangle$ and the branching ratio for $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay, when both leptons are longitudinally polarized

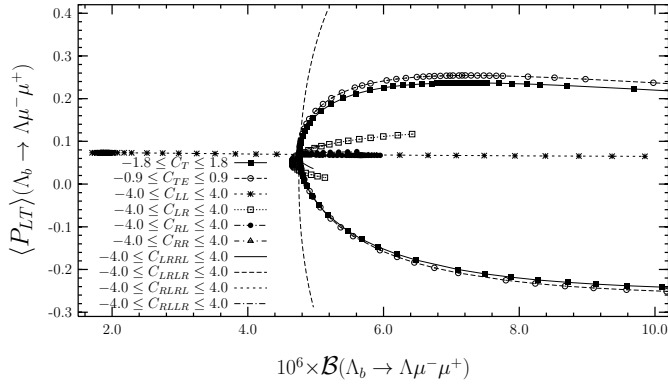


Fig. 2. The same as in Fig. 1, but for the averaged double-lepton polarization asymmetry $\langle P_{LT} \rangle$

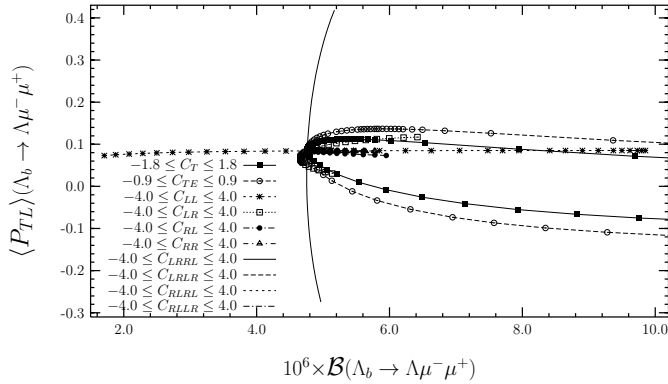


Fig. 3. The same as in Fig. 1, but for the averaged double-lepton polarization asymmetry $\langle P_{TL} \rangle$

$\langle P_{LT} \rangle$ exceeds the SM prediction three times and $\langle P_{TL} \rangle$ two times, respectively. More essential than that, $\langle P_{LT} \rangle$ as well as $\langle P_{TL} \rangle$ both change their signs when Wilson coefficients vary in the allowed region. Such behaviors can serve as a good test for establishing new physics beyond the SM. (3) $\langle P_{NN} \rangle$ and $\langle P_{TT} \rangle$ are quite sensitive to the existence of the vector interactions, especially to C_{LR} . In the presence

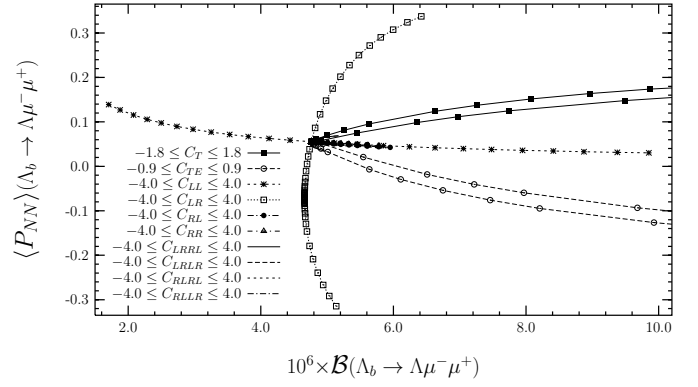


Fig. 4. The same as in Fig. 1, but for the averaged double-lepton polarization asymmetry $\langle P_{NN} \rangle$

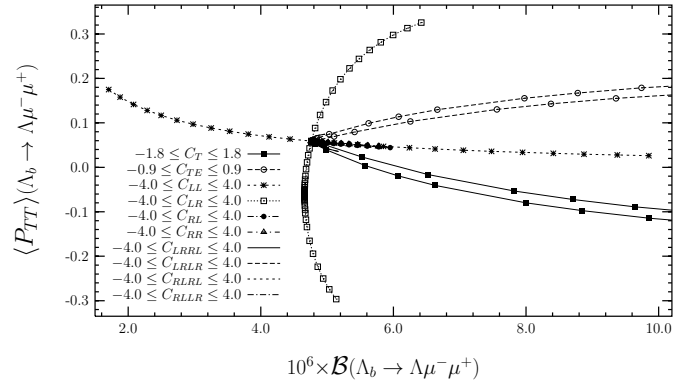


Fig. 5. The same as in Fig. 1, but for the averaged double-lepton polarization asymmetry $\langle P_{TT} \rangle$

of this coefficient, the values of $\langle P_{NN} \rangle$ and $\langle P_{TT} \rangle$ can both exceed the SM result three–four times, and both change their sign when C_{LR} varies in the allowed region. Therefore, determination of the sign and magnitude of $\langle P_{NN} \rangle$ and $\langle P_{TT} \rangle$ can give unambiguous information about the existence of the new vector type interaction.

We do not present the correlation of $\langle P_{LN} \rangle$, $\langle P_{NL} \rangle$, $\langle P_{NT} \rangle$ and $\langle P_{TN} \rangle$ on the branching ratio, since their values are quite small.

In Figs. 6–12 the correlations of $\langle P_{ij} \rangle$ with the branching ratios for the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay are presented. Similar to the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay, we observe that several of the double-lepton polarization asymmetries are very sensitive to the existence of new physics and the presence of the new Wilson coefficients can produce results that depart considerably from the SM prediction. More precisely, we can briefly comment on the results as follows.

- (1) All $\langle P_{ij} \rangle$ are very sensitive to the existence of tensor interactions with coefficients C_T and C_{TE} .
- (2) The situation for the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay is slightly different from the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay, for which a considerable dependence on the existence of the scalar type interaction is observed, especially for the coefficients C_{LRLR} and C_{RLLR} .

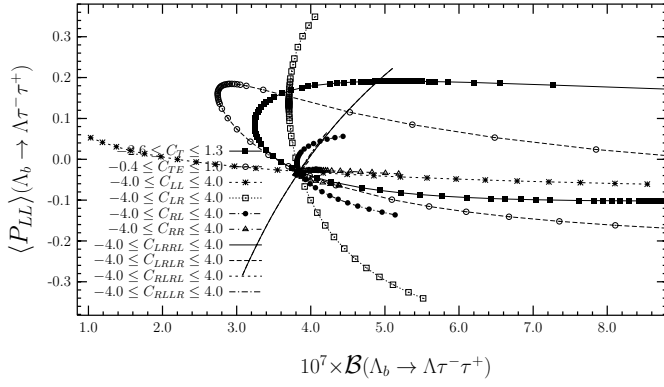


Fig. 6. The same as in Fig. 1, but for $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay

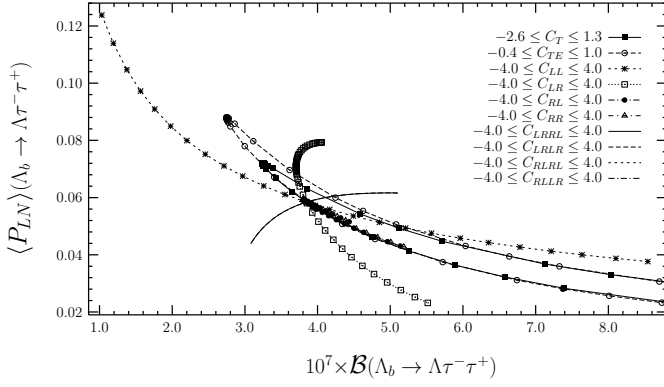


Fig. 7. The same as in Fig. 6, but for the averaged double-lepton polarization asymmetry $\langle P_{LN} \rangle$

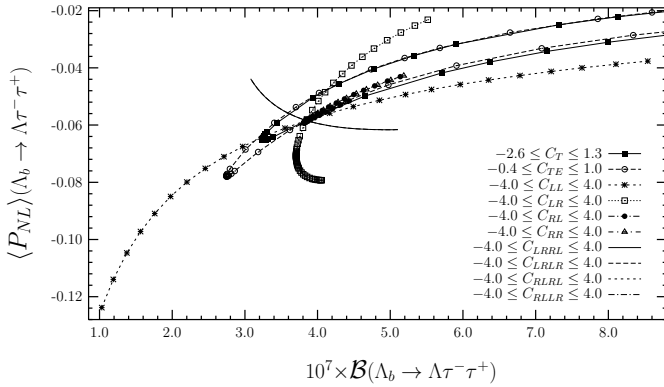


Fig. 8. The same as in Fig. 6, but for the averaged double-lepton polarization asymmetry $\langle P_{NL} \rangle$

(3) $\langle P_{LL} \rangle$, $\langle P_{LT} \rangle$, $\langle P_{TL} \rangle$ and $\langle P_{TT} \rangle$ exhibit a strong dependence on the vector interaction with coefficients C_{LR} , while $\langle P_{LN} \rangle$ and $\langle P_{NL} \rangle$ do so on the vector interaction with coefficient C_{LL} .

(4) $\langle P_{LL} \rangle$ and $\langle P_{NN} \rangle$ change sign when the new Wilson coefficients vary in the allowed region. For this reason, measurement of magnitude and sign of the averaged double-lepton polarization asymmetry can be a very useful tool in establishing new physics beyond the SM.

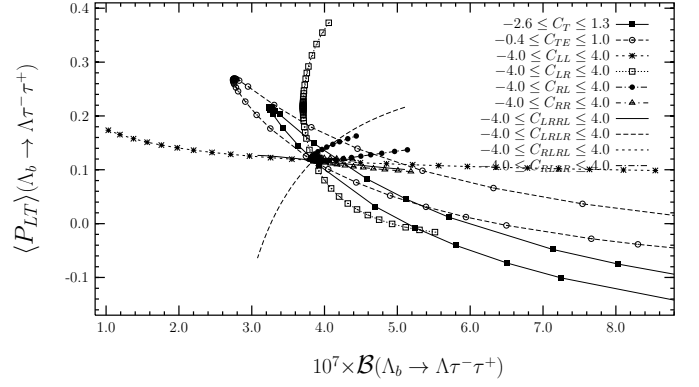


Fig. 9. The same as in Fig. 2, but for $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay

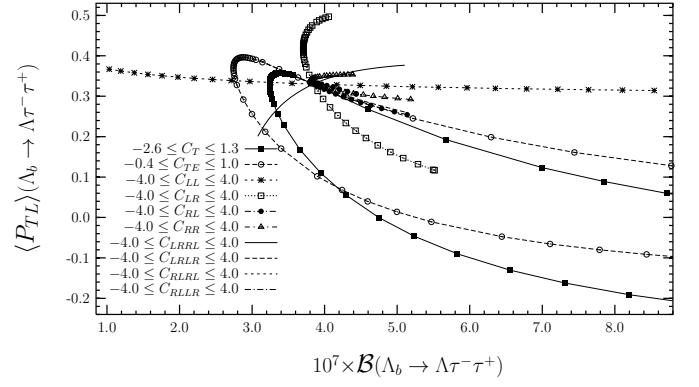


Fig. 10. The same as in Fig. 3, but for $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay

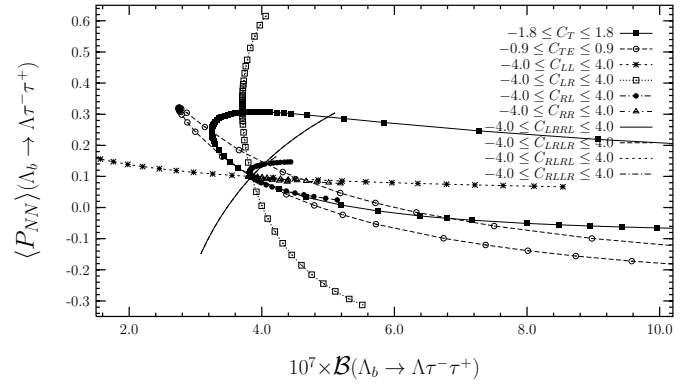


Fig. 11. The same as in Fig. 4, but for $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay

The next question we would like to discuss is the possibility of the existence of such a region of the new Wilson coefficients for which the branching ratio coincides with SM result, while the double-lepton polarizations do not, similar to the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay.

From the relevant figures we see that the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay is far more informative on this issue. There exist regions of the new Wilson coefficients C_{LR} , C_T and C_{TE} , where double-lepton polarization asymmetries differ from the SM result, but the branching ratio coincides with that of the SM.

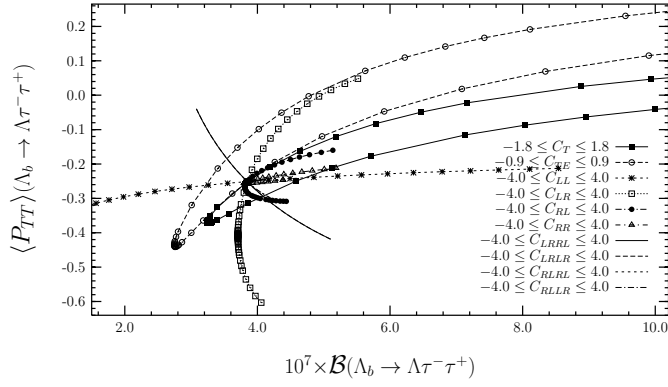


Fig. 12. The same as in Fig. 10, but for the averaged double-lepton polarization asymmetry $\langle P_{TT} \rangle$

At the end of this section, let us discuss the problem of measurement of the lepton-polarization asymmetries in experiments. Experimentally, to measure an asymmetry $\langle P_{ij} \rangle$ of the decay with the branching ratio \mathcal{B} at $n\sigma$ level, the required number of events (i.e., the number of $B\bar{B}$ pairs) are given by the expression

$$N = \frac{n^2}{\mathcal{B}s_1s_2\langle P_{ij} \rangle^2},$$

where s_1 and s_2 are the efficiencies of the leptons. Typical values of the efficiencies of the τ -leptons range from 50% to 90% for their various decay modes (see for example [25] and references therein), and the error in τ -lepton polarization is estimated to be about (10 ÷ 15)% [26]. As a result, the error in measurement of the τ -lepton asymmetries is of the order of (20 ÷ 30)%, and the error in obtaining the number of events is about 50%.

From the expression for N we see that, in order to observe the lepton-polarization asymmetries in $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ and $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decays at 3σ level, the minimum number of required events are (for the efficiency of τ -lepton we take 0.5):

(1) for $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay

$$N = \begin{cases} 2.0 \times 10^6 & \text{(for } \langle P_{LL} \rangle), \\ 2.0 \times 10^8 & \text{(for } \langle P_{LT} \rangle, \langle P_{TL} \rangle, \langle P_{NN} \rangle, \langle P_{TT} \rangle), \end{cases}$$

(2) for $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay

$$N = \begin{cases} (4.0 \pm 2) \times 10^9 & \text{(for } \langle P_{LT} \rangle, \langle P_{NN} \rangle), \\ (1.0 \pm 0.5) \times 10^9 & \text{(for } \langle P_{TT} \rangle), \\ (2.0 \pm 1.0) \times 10^{11} & \text{(for } \langle P_{LN} \rangle, \langle P_{NL} \rangle), \\ (9.0 \pm 4.5) \times 10^8 & \text{(for } \langle P_{TL} \rangle). \end{cases}$$

The numbers of $B\bar{B}$ pairs that are produced at B -factories and LHC are about $\sim 5 \times 10^8$ and 10^{12} , respectively. As a result of a comparison of these numbers and N , we conclude that only $\langle P_{LL} \rangle$ in the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay and $\langle P_{LT} \rangle$, $\langle P_{NN} \rangle$ and $\langle P_{TL} \rangle$ in the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay, can be detectable at LHC.

In summary, we present the most general analysis of the double-lepton polarization asymmetries in the $\Lambda_b \rightarrow$

$\Lambda \ell^+ \ell^-$ decay using the most general, model independent form of the effective Hamiltonian. The correlation of the averaged double-lepton polarization asymmetries on the branching ratio have been studied. Our results show that the study of double-lepton polarization asymmetries can serve as a good test for establishing new physics beyond the SM. Moreover, we observe that there exist regions of the new Wilson coefficients for which double-lepton polarization asymmetries depart considerably from the SM, while the branching ratio coincides with that of the SM predictions.

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